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Report on the doctoral dissertation
Feedback Linearization of Mechanical Control Systems
by
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1. Introduction

The reported dissertation addresses the problem of linearization of a mechanical control system by a mechanical state space transformation or a mechanical feedback, in the context of analysis and control of robotic systems. The subject of this dissertation belongs to the field of Engineering and technology, discipline Automation, electronic and electrical engineering (former Automation and robotics). The dissertation presents an example of a successful application of geometric methods based on the Riemannian geometry to engineering problems of practical significance, From a methodological point of view the dissertation refers to the tradition established in the years 1970-1980s by Krener, Brockett, Sussmann, Jakubczyk, Respondek, and others, concerned with converting a nonlinear control system to a linear system by either state or feedback transformations. It goes without saying that the paradigm of linearization has opened a new research perspective in nonlinear control theory, that resulted both in a remarkable progress in the theory of nonlinear control systems, as well as in the design of new control algorithms for systems important from the viewpoint of applications, including those belonging to robotics. I do agree with a kind of poetic assertion on p. 1 of the disertation that **fruitful results of this paradigm are consumed to these days**. Specifically, in recent years demanding applications in robotics have become a driving force in the development of a theory of mechanical control systems.

This dissertation makes a contribution to the field of control of mechanical robotic systems. By introducing the concept of a mechanical control system and a mechanical transformations the dissertation has provided an original solution to the problem of equivalence of mechanical control systems realized by mechanical state or feedback transformation, in particular, of the equivalence of a mechanical control system to a linear mechanical control system. The roots of this dissertation can be tracked back to the doctoral dissertation of Dr S. Ricardo, entitled **Geometry of mechanical control systems with applications to systems subject to second-order nonholonomic constraints**, defended at INSA Rouen in 2008. Dr Ricardo studied the question that may be paraphrased as "what makes a control system mechanical?", and provided conditions for the linearization of a mechanical system by the state space diffeormorphism. Although the reported dissertation also touches the problem of the state space linearization, its main contribution lies in the linearization of mechanical control systems by the mechanical feedback. A novelty going beyond the traditional approach to the lienarization consists in the admission of non-controllable linear systems as the target of the linearization. Theoretical results obtained in the dissertation have been illustrated by means of examples of mechanical robotic systems, and then used in order to design control algorithms in accordance with the two stage paradigm: linearization, and linear control. Summarizing, the dissertation has shown that **It is possible to formulate a theory of mechanical feedback linearization of mechanical control systems using techniques of geometric control theory and differential geometry.**

2. Composition and content

The dissertation has been written in English. Its main body includes 135 pages of text, preceded by Acknowledgements, summaries in English, Polish, and French, a list of figures, and a list of symbols. The main text is divided into 8 chapters and ended with a bibliography with 47 items. Chapter 1 presents shortly the background, motivation and the outline of the dissertation as well as states the research hypothesis: **It is possible to formulate a theory of mechanical feedback linearization of mechanical control systems using techniques of geometric control theory and differential geometry.** Chapter 2 introduces fundamental geometrical concepts underlying the dissertation, with emphasis on the Riemannian geometry, as well as presents some basics of the theory of control affine systems. The next chapter 3 defines the mechanical control system and the Lagrangian mechanical system, and delivers relevant descriptions as mechanical control systems of 4 mechanical robotic systems: robotic manipulators with rigid and flexible joints, the inertia wheel pendulum, and the so called TORA system. Chapter 4 of the dissertation has been devoted to linear mechanical control systems. That chapter contains a definition of the linear mechanical control and linear Lagrangian mechanical systems, and of the linear mechanical feedback, culminating with an equivalent of the Brunovsky normal form theorem for linear mechanical systems. The main linearization results have been provided in chapters 5 and 6. Chapter 5 deals with the mechanical state space linearization. This chapter includes new necessary and sufficient conditions for the mechanical state space linearization under assumption that the target linear mechanical control system is controllable (complementing the results of S. Ricardo). Moreover, a completely new research avenue is explored of the linearization to non-controllable linear systems. Chapter 6 distinguishes by abundance of new concepts and results. Beginning with a definition of the mechanical feedback equivalence, this chapter gives necessary and sufficient conditions for the mechanical feedback linearization of a single-input mechanical control system to a controllable linear system, for the feedback input-output linearization, for the feedback transformation of a multiple input system to a non-controllable linear system, and closes with the statement of linearizability conditions for mechanical control systems devoid of potential forces. In chapter 7 various linearizability conditions are applied to mechanical robotic systems like the robot car propelled by a blower and the cart-pole system, then the linearization method is used in the design of control algorithms for stabilization, motion planning, and trajectory tracking of systems dealt with in chapter 3. The dissertation is concluded with chapter 8.

3. Assessment

Besides the mentioned dissertation of S. Ricardo and the following publications [36-39] this dissertation draws upon the monograph by F. Bullo and A. D. Lewis **Geometric Control of Mechanical Systems**, Springer, 2005, and the chapter by M. W. Spong **On feedback linearization of robot manipulators and Riemannian curvature**, published in **Essays on Mathematical Robotics**, Springer, 1998. As has been mentioned above, this dissertation brings in two kinds of contributions to the control of mechanical robotic systems: a cognitive one and an enginering-oriented. The cognitive contribution not only offers a novel understanding of mechanical control systems, but also lays solid mathematical foundations for engineering applications. Moreover, although clearly inspired by engineering problems, this dissertation goes far beyond the judgement **mathematica ancilla technologiae**, so its cognitive results are charming on their own. The main results of this dissertation can be summarized in the following way:

1. Definition of basic concepts related to mechanical control systems and their mechanical equivalence (state space, feedback) embedded in the Riemannian geomtery,
2. Establishment of the equivalent of the Brunovsky classification theorem for linear mechanical control systems (Theorem 4.6)

3. Derivation and proof of necessary and sufficient conditions for the local mechanical state space feedback linearization targeted at either controllable or non-controllable linear systems (Theorems 5.5 and 5.6)
4. Establishment of necessary and sufficient conditions for the local linearization by mechanical feedback of single-input mechanical systems (Theorem 6.11 and Proposition 6.12) and for the input-output linearization (Proposition 6.15 and Theorem 6.18) to a controllable linear system
5. Derivation and proof of necessary and sufficient conditions for the local mechanical feedback linearization to a non-controllable linear system (Theorem 6.21)
6. Establishment of linearizability conditions for mechanical control systems free of potential forces Corollary 6.24 and Proposition 6.25)
7. Description of a number of representative mechanical robotic systems as mechanical control systems, checking linearizability conditions, and appliction of the linearizability property to the design of control algorithms for stabilization, motion planning, and trajectory tracking of these systems (chapter 7).

I appreciate very much the outstanding quality of this dissertation as well as the expertise of its Author in the area of differential geometry, control theory, and robotics. The dissertation contains proofs of 20 lemmas, propositions and theorems, some of them long and technical, that have been provided with remarkable mathematical skill. At the same time, solutions of control problems presented in chapter 7 reveal the control engineering profesionalism of the Author. Last but not least, the composition of the dissertation is reader-friendly: the theoretical chapters of increasing mathematical demand with culmination in chapter 6 smoothly relieve in the long and insightful chapter 7 devoted to engineering applications.

4. Debatable remarks and suggestions for further research

In my opinion, some issues addressed in the dissertation may need a further clarification and/or may suggest some areas for further research. They include the following:

- I do appreciate the cognitive significance of the problem of linearization of mechanical system to a linear mechanical system. But, if the linearization is to be used just in order to find a control algorithm for a mechanical system then the requirement that the resulting linear system needs to be mechanical (not just linear) seems to be not necessary. Is there any engineering motivation behind this requirement?
- A novelty of this dissertaion consists in considering the lineariztion to a non-controllable linear system. This is a challenging subject mathematically. However, if a linear system as a sort of the normal form of a non-linear system is to be used for control purposes, we need to know control algorithms for the normal form. How to control a linear system that is not controllable? Is it possible that the linearization to a non-controllable liear system has primarily an aesthetic appeal?
- A crucial point in using the linearization method in the design of control algorithms is the explicit computation of the state space/feedback transformations. This is usually a hard problem, and often these transformations cannot be found in the analytic form. Even if they can be found, they may appear to be locally defined and complex. What can be said about computability of the mechanical (state space, feedback) transformations?
- Two primary results of the dissertation (Theorem 6.11 and Proposition 6.12) refer to single-input mechanical systems. Is there any essential difficulty with generalizing these results to multi-input systems?
- The control problems studied in chapter 7 refer exclusively to mechanical systems with 2 dof and 1 control for which the linearizing feedback can be found either via a linearizing output (inertia wheel pendulum) or, likely, by inspection (TORA, flexible manipulator link). Has the Author considered more complex systems, specifically systems for which no control algorithms are known and the use of linearization is the only solution?
- I think that the overview presented in the summaries of the dissertation has been aimed mostly at providing the reader with a flavor of the obtained results. They cannot be fully appreciated without definitions of the mathematical notations used there. In this contex the list of symbols on p. xix does not look complete.

- In chapter 3 the mechanical control systems are distinguished from the Lagrangian mechanical control systems. Could, possibly, the Author show an example of a mechanical system that is not Lagrangian?
- In the similar spirit: In analytic mechanics the Lagrangian formalism is equivalent to the Hamiltonian one. What is the reason of preferring in the disertation the former over the latter?
- The mechanical control systems considered in this dissertation do not include the dynamics of non-holonomic systems. Is it possible to extend the obtained results accordingly?
- A related paper to this dissertation, entitled **Constructive feedback linearization of underactuated mechanical systems with 2-DOF** by Acosta and Lopez-Martinez, was presented at 44 CDC and ECC in 2005, in Sevilla. Specifically, the stabilization problem of the inertia wheel pendulum was studied there as an example. Focussing just on this example, what are the advantages of the approach fostered in this dissertation?
- Is it true, as suggested on p. 26 and in identity (5.6), that the projection of a system trajectory depends only on the projected initial state?
- How do we know that the normalized form (6.11) is controllable, and it what sense?
- Theorem 5.6 on the state space linearization suggests a neat intuitive interpretation (zero Christoffel symbols, constatnt control vector fields and linear potential term). What is the intuitive meaning of conditions for the feedback linerization provided by Theorem 6.21?
- The List of symbols would benefit form being more complete and form displaying the number of page on which a given symbol has been introduced/defined.
- It is quite unfavorable that no publication of the dissertation's Author has been mentioned in the bibliography.

5. Conclusion

In my opinion, the dissertation reported here has made a substantial contribution to the development of mathematical methods of robotics, specifically to the new field of feedback linearization of mechanical robotic systems. Having a fundamental significance to the understanding of mechanical control systems, the results reported in this dissertation play essential role in the control engineering of robotic systems. On account of both the substantive quality as well as the presentation merits of the dissertation, my overall assessment of this dissertation is excellent. **In conclusion, I'm fully convinced that the doctoral dissertation of Mr. Marcin Nowicki entitled Feedback Linearization of Mechanical Control Systems satisfies all academic requirements referring to the doctoral dissertations in the field of Engineering and technology, discipline automation, electronic and electrical engineering, and I firmly recommend its admission to the public defense.**